Mensuration

Practice Set 7.1

Q. 1. Find the volume of a cone if the radius of its base is 1.5 cm and its perpendicular height is 5 cm.

Answer : Radius of base of cone, r = 1.5cm

Perpendicular height of cone, H = 5cm

As we know that,

Volume of the cone, $V = \frac{1}{3}\pi r^2 H$

On substituting the given values,

$$\Rightarrow V = \frac{1}{3} \times \frac{22}{7} \times (1.5)^2 \times 5$$
$$= \frac{1}{3} \times \frac{22}{7} \times 2.25 \times 5$$
$$= \frac{247.5}{21}$$

⇒ V = 11.79 cm^3

∴ Volume of the cone is 11.79 cm^3

Q. 2. Find the volume of a sphere of diameter 6 cm.

Answer : Diameter of the sphere, d = 6 cm

⇒ Radius of sphere, r = 3 cm

As we know, the volume of sphere, $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \frac{4}{3} \times \frac{22}{7} \times (3)^3$$

⇒ V = 113.04 cm^3

∴ Volume of sphere is 113.04 cm^3

Q. 3. Find the total surface area of a cylinder if the radius of its base is 5 cm and height is 40 cm.

Answer : Radius of base of cylinder, r = 5 cm

Height of cylinder, H = 40 cm

As we know,

Total surface area of cylinder, A = 2πr (r + h)

On substituting the values, we get,

A = 2× (3.14) × 5 × (5 + 40)

⇒ A = 1413 sq.cm

∴ Total surface area of square is 1413 sq. cm

Q. 4. Find the surface area of a sphere of radius 7 cm.

Answer : Radius of sphere, r = 7cm

As we know, the surface area of sphere, A = $4\pi r^2$

On substituting the values, we get,

$$A = 4 \times \frac{22}{7} \times 7^2$$

⇒ A = 616 sq. cm

∴ Surface area of sphere is 616 sq. cm

Q. 5. The dimensions of a cuboid are 44 cm, 21 cm, 12 cm. It is melted and a cone of height 24 cm is made. Find the radius of its base.

Answer : Since the volume of cuboid = length× breadth× height

⇒ Volume of cuboid = Product of the given three dimensions

⇒ Volume of cuboid = 44× 21× 12

Height of cone, H = 24 cm

Let Radius of cone be r

Volume of cone $= \frac{1}{3}\pi r^2 H$

As the cone is melted to form a cone,

⇒ Volume of cone = volume of cuboid

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 24 \times r^2 = 44 \times 21 \times 12$$

$$r^2 = \frac{44 \times 21 \times 12 \times 7 \times 3}{22 \times 24}$$

$$r^2 = 21 \times 7 \times 3$$

$$r^2 = 21 \times 21$$

$$\Rightarrow r = 21 \text{ cm}$$

⇒ r = 21 cm

∴ The radius of the cone is 21 cm

Q. 6. Observe the measures of pots in figure 7.8 and 7.9. How many jugs of water can the cylindrical pot hold?

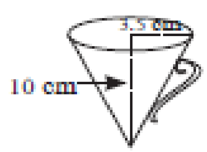


Fig 7.8
conical water jug

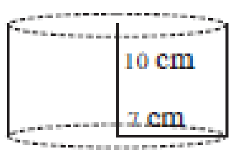


Fig 7.9
cylindrical water pot

Answer : Height of water jug, H_J = 10cm

Radius of water jug, R_J = 3.5 cm

Volume of conical jug, $V_J = \frac{1}{3}\pi (R_j)^2 H_J$

Let the number of jugs be n.

Height of cylindrical pot, H_P = 10 cm

Radius of pot, R_P = 7 cm

$$\text{Volume of pot} = \pi(R_P)^2 H_P$$

Since the water is transferred from pot to ‘n’ number of jugs,

⇒ Volume of pot = n × Volume of jug

$$\Rightarrow \pi(R_P)^2 H_P = n \times \frac{1}{3}\pi(R_j)^2 H_J$$

On substituting the given values,

$$\Rightarrow n = 3 \times \left(\frac{R_P}{R_J}\right)^2 \times \frac{H_P}{H_J}$$

⇒ n = 3× 2^2× 1

⇒ n = 12 cm

∴ The cylindrical pot can hold 12 jugs of water.

Q. 7. A cylinder and a cone have equal bases. The height of the cylinder is 3 cm and the area of its base is 100 cm^2.The cone is placed upon the cylinder. Volume of the solid figure so formed is 500 cm^3. Find the total height of the figure.

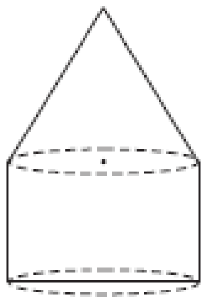


Fig 7.10

Answer : Let the radius of base be r.

Let the height of cone = H

Height of cylinder, h = 3cm

Area of base, A = 100 sq. cm

As we know the area of circle is πr^2

$\Rightarrow \pi r^2 = 100$ ….. (1)

Volume of complete solid figure, V = Volume of cone + volume of cylinder

$$\Rightarrow V = \frac{1}{3}\pi r^2 H + \pi r^2 h$$

$$\Rightarrow V = \pi r^2 \left(h + \frac{H}{3}\right)$$

It is given that volume of solid figure, V = 500 cubic cm

On substituting the value of V and πr^2 from eq (1), we get,

$$\Rightarrow 500 = 100(3 + \frac{H}{3})$$

$\Rightarrow$ H = 6 cm

Total height of figure = h + H = 3 + 6 = 9 cm

∴ Total height is 9 cm

Q. 8. In figure 7.11, a toy made from a hemisphere, a cylinder and a cone is shown. Find the total area of the toy.

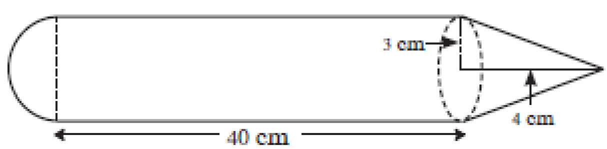


Fig. 7.11

Answer : Radius of circular region, r = 3cm

Height of Cylinder, H = 40cm

Height of cone, h = 4cm

As we know,

Surface area of hemisphere, $A_H = 2\pi r^2$

Surface area of cylinder, $A_{cy} = 2\pi rH$

Surface area of cone is,

$$A_{CO} = \pi r\sqrt{r^2 + h^2}$$

$$\Rightarrow A_{CO} = \pi r\sqrt{3^2 + 4^2}$$

$A_{CO} = 5\pi r$

Total area = $A_H + A_{CY} + A_{CO}$

On substituting the values, we get,

⇒ Total area = $\pi r(2r + 2H + 5)$

⇒ Total area = $\pi \times 3 \times (6 + 80 + 5)$

⇒ Total area = 273π

∴ Total area of the figure is 273π sq. cm

Q. 9. In the figure 7.12, a cylindrical wrapper of flat tablets is shown. The radius of a tablet is 7 mm and its thickness is 5 mm. How many such tablets are wrapped in the wrapper?

Fig. 7.12

Answer : Since there are only flat tablets which are stacked onto each other inside the cylindrical wrapper, so the radius of the wrapper does not matter.

Height of wrapper, H = 10cm = 100mm

Thickness of the tablet, h = 5mm

$$\text{Number of tablets} = \frac{\text{Height of wrapper}}{\text{Thickness of the tablet}}$$

⇒ Number of the tablets = 20

∴ There are 20 tablets inside the cylindrical wrapper.

Q. 10. Figure 7.13 shows a toy. Its lower part is a hemisphere and the upper part is a cone. Find the volume and the surface area of the toy from the measures shown in the figure.(π= 3.14)

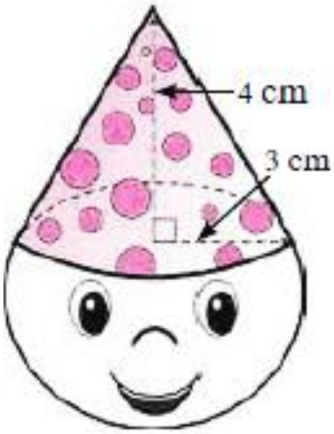


Fig. 7.13

Answer : Radius of the circular base, r = 3cm

Height of cone, h = 4cm

Volume of cone, $V_C = \frac{1}{3}\pi r^2 h$

Volume of sphere, $V_S = \frac{2}{3}\pi r\text{^}3$

Volume of toy, $V = \frac{1}{3}\pi r^2(h + 2r)$

$\Rightarrow V = \frac{1}{3} \times (3.14) \times 3^2 \times (4 + 6)$

⇒ V =3.14×3 × 10

⇒ V = 94.20 cubic cm

Now,

Surface area of cone, $S_C = \pi r\sqrt{r^2 + h^2}$

$\Rightarrow S_C = (3.14) \times 3 \times \sqrt{3^2 + 4^2}$

$\Rightarrow S_C = 3.14 \times 3 \times 5$

$\Rightarrow S_C = 47.1$ sq.cm

Surface area of sphere, $S_S = 2\pi r^2$

$\Rightarrow S_S = 2 \times (3.14) \times 3^2$

$\Rightarrow S_S = 56.52$ sq.cm

Surface area of toy = 47.1 + 56.52 = 103.52 sq.cm

∴ Volume and surface area of toy is 94.20 cm^3 and 103.52 cm^2 respectively

Q. 11. Find the surface area and the volume of a beach ball shown in the figure.

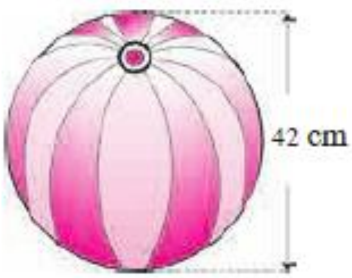


Fig. 7.14

Answer : Diameter of spherical ball, d = 42 cm

Radius of ball, r =21 cm

As we know that, Surface area of sphere, $A = 4\pi r^2$

$\Rightarrow A = 4\times (3.14) \times (21)^2$

$\Rightarrow A = 5538.96$ sq.cm

Also,

$$\text{Volume of sphere, } V = \frac{4}{3}\pi r^3$$

$$\Rightarrow V = \frac{4}{3} \times (3.14) \times (21)^3$$

$\Rightarrow$ V = 38772.72 cm^3

∴ The surface area and volume of the spherical ball is 5538.96 cm^2 and 38772.72 cm^3 respectively

Q. 12. As shown in the figure, a cylindrical glass contains water. A metal sphere of diameter 2 cm is immersed in it. Find the volume of the water.

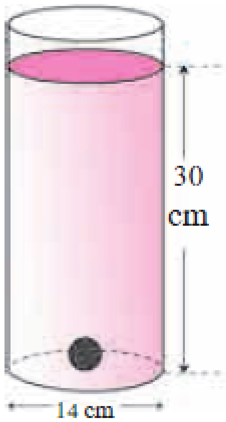


Fig. 7.15

Answer : Diameter of glass, D = 14 cm

$\Rightarrow$ Radius of glass, r = 7cm

Height of water level, H = 30 cm

We know that, volume of cylinder, V = $\pi R^2 H$

$\Rightarrow V = \pi \times (7)^2 \times 30$

$\Rightarrow$ V = 1470π cubic cm

Diameter p of metal sphere, d = 2 cm

Radius of metal sphere, r = 1cm

As we know that,

$$\text{Volume of metal sphere, } v = \frac{4}{3}\pi r^3$$

$$\Rightarrow v = \frac{4}{3} \times \pi \times 1^3$$

⇒ v = 1.33π cubic cm

Volume of water = Volume of glass – Volume of metal sphere

⇒ Volume of water = 1470π – 1.33π

∴ Volume of the water is 1468.67π cm^3

Practice Set 7.2

Q. 1. The radii of two circular ends of frustum shape bucket are 14 cm and 7 cm. Height of the bucket is 30 cm. How many liters of water it can hold? (1 litre = 1000cm^3)

Answer : The two radii of frustum are, r_1 = 14cm and r_2 = 7cm

Height of bucket, H = 30 cm

As we know that,

$$\text{Volume of frustum, } V = \frac{1}{3}\pi H(r_1^2 + r_2^2 + r_1 r_2)$$

$$\Rightarrow V = \frac{1}{3} \times \frac{22}{7} \times 30 \times (14^2 + 7^2 + 14 \times 7)$$

⇒ V = 10780 cm^3

Now, as 1litre = 1000cm^3

⇒ V = 10.780 litre

∴ Bucket can hold 10.780 litres of water

Q. 2. The radii of ends of a frustum are 14 cm and 6 cm respectively and its height is 6 cm. Find its

i) Curved surface area
ii) Total surface area
iii) Volume

Answer : (i) The two radii of frustum are, r_1 = 14cm and r_2= 6cm

Height of frustum, H = 6cm

Slant height of frustum, $l = \sqrt{H^2 + (r_1 - r_2)^2}$

$\Rightarrow l = \sqrt{6^2 + (14-6)^2}$

$\Rightarrow l = \sqrt{36 + 64}$

⇒ l = 10

As we know that,

Curved surface area, $A_C = \pi l(r_1 + r_2)$

⇒ A_C = (3.14) × 10 × (14 + 6)

⇒ A_C = (3.14) × 200

⇒ A_C = 628 sq. cm

∴ the curved surface area of frustum is 628 sq. cm

(ii) Total surface area, A_T = Curved surface area + area of the two circular regions

$A_T = A_C + \pi r_1^2 + \pi r_2^2$

On substituting the above values, we get,

⇒ $A_T = 628 + (3.14) \times (14^2 + 6^2)$

⇒ A_T = 628 + (3.14)× (196 + 36)

⇒ A_T = 628 + 728.48

⇒ A_T = 1356.48 sq. cm

∴ the total surface area of frustum is 1356.48 cm^2

(iii) As we know,

$$\text{The Volume of frustum, } V = \frac{1}{3}\pi H(r_1^2 + r_2^2 + r_1 r_2)$$

On substituting the values, we get,

$$V = \frac{1}{3} \times 3.14 \times 6 \times (14^2 + 6^2 + 14 \times 6)$$

$$V = \frac{1}{3} \times 3.14 \times 6 \times (196 + 36 + 84)$$

$$V = \frac{1}{3} \times 3.14 \times 6 \times 316$$

V = 1984.48 cm^3

∴ Volume of the frustum is 1984.48 cm^3

Q. 3. The circumferences of circular faces of a frustum are 132 cm and 88 cm and its height is 24 cm. To find the curved surface area of the frustum complete the following activity. $\left(\pi = \frac{22}{7}\right)$.

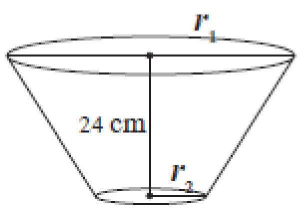


Fig. 7.23

$\text{Circumference}_1 = 2\pi r_1 = 132$

$$r_1 = \frac{132}{2\pi} = \square$$

$\text{Circumference}_2 = 2\pi r_2 = 88$

$$r_2 = \frac{88}{2\pi} = \square$$

Slant height of frustum, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$$= \sqrt{\square^2 + \square^2}$$

$$= \square \text{ cm}$$

Curved Surface area of frustum = $\pi(r_1 + r_2)l$

$$= \pi \times \square \times \square$$

$$= \square \text{ sq. cm.}$$

Answer : $\text{Circumference}_1 = 2\pi r_1 = 132$

$$\Rightarrow r_1 = \frac{132}{2\pi} = 21$$

$\text{Circumference}_2 = 2\pi r_2 = 88$

$$\Rightarrow r_2 = \frac{88}{2\pi} = 14$$

Slant height of frustum, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$$l = \sqrt{24^2 + (21 - 14)^2}$$

$$l = \sqrt{24^2 + 7^2}$$

$$l = \sqrt{576 + 49}$$

$$l = 25 \text{ cm}$$

Curved Surface area of frustum = $\pi(r_1 + r_2)l$

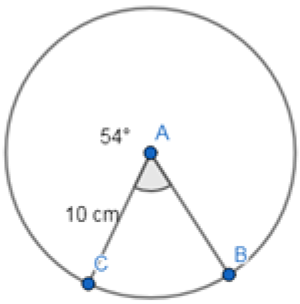
⇒ Curved Surface Area of frustum = π× (21 + 14) × 25

⇒ Curved surface area = π × 35 × 25 = 2750 sq. cm

Practice Set 7.3

Q. 1. Radius of a circle is 10 cm. Measure of an arc of the circle is 54°. Find the area of the sector associated with the arc. (π = 3.14)

Answer : Radius of circle, r = 10 cm



Angle made between the arc, θ = 54°

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

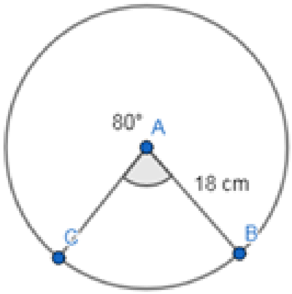
$$\Rightarrow \text{Area of sector} = \frac{54}{360} \times 3.14 \times 10^2$$

⇒ Area of sector = 47.1 sq. cm

Q. 2. Measure of an arc of a circle is 80 cm and its radius is 18 cm. Find the length of the arc. (π = 3.14)

Answer : Measure of an arc of circle, θ = 80°

Radius of circle, r = 18 cm



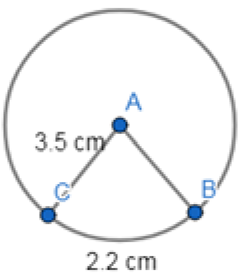
$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow \text{Length of arc} = \frac{80}{360} \times 2 \times 3.14 \times 18$$

⇒ Area of sector = 25.12 cm

Q. 3. Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector.

Answer :



Radius of circle, r = 3.5 cm

Length of arc, l = 2.2 cm

As we know,

$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow 2.2 = \frac{\theta}{360} \times 2 \times 3.14 \times 3.5$$

$$\Rightarrow \theta = 36.03$$

Also,

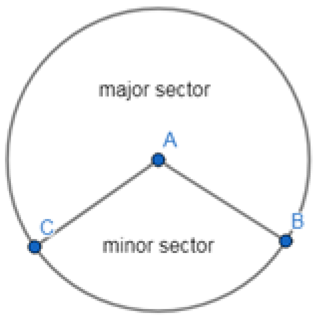
$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow \text{Area} = \frac{36.03}{360} \times 3.14 \times (3.5)^2$$

⇒ Area of sector = 3.85 sq. cm

Q. 4. Radius of a circle is 10 cm. Area of a sector of the sector is 100 cm^2. Find the area of its corresponding major sector. (π = 3.14)

Answer : Radius of circle, r = 10 cm



Area of sector (minor sector) = 100 sq. cm

Area of circle, $A_C = \pi r^2$

⇒ $A_C = 3.14 \times 10^2$

⇒ A_C = 314 sq. cm

Area of major sector, A_M = area of circle – area of minor sector

⇒ $A_M = A_C - 100$

⇒ A_M = 314 -100 = 214 sq. cm

∴ Area of major sector is 214 sq. cm

Q. 5. Area of a sector of a circle of radius 15 cm is 30 cm^2. Find the length of the arc of the sector.

Answer : Radius of circle, r = 15 cm

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow 30 = \frac{\theta}{360} \times 3.14 \times r^2$$

$$\Rightarrow \theta = \frac{360 \times 30}{3.14 \times 15^2}$$

Also,

$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

On substituting the values, we get,

$$\text{Length of arc} = \frac{30}{3.14 \times 15^2} \times 2\pi r$$

⇒ Length of arc = 4 cm

Q. 6. In the figure 7.31, radius of the circle is 7 cm and m(arc MBN) = 60°,find

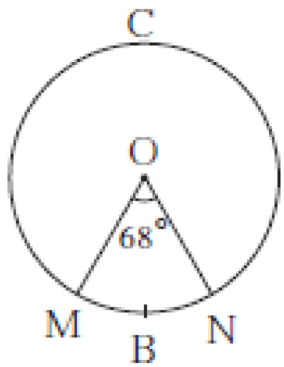


Fig. 7.31

(1) Area of circle
(2) A(O – MBN)
(3) A(O - MCN)

Answer : (1) Radius of circle, r = 7cm

Area of circle, $A_C = \pi r^2$

$$\Rightarrow A_C = \frac{22}{7} \times 7^2$$

⇒ A_C = 154 sq. cm

∴ Area of circle is 154 sq.cm

(2) Angle subtended by the arc = 60°

As we know,

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow \text{Area }(O - MBN) = \frac{60}{360} \times \text{Area of circle}$$

$$\Rightarrow \text{Area }(O - MBN) = \frac{1}{6} \times A_C$$

⇒ A(O- MBN) = 25.7 sq. cm

(3) A(O- MCN) = Area of circle – A(O – MBN)

⇒ Area(O – MCN) = A_C – 25.7

⇒ Area(O – MCN) = 154 – 25.7

∴ Area of major sector is 128.3 sq. cm

Q. 7. In figure 7.32, radius of circle is 3.4 cm and perimeter of sector P-ABC is 12.8 cm. Find A (P-ABC).

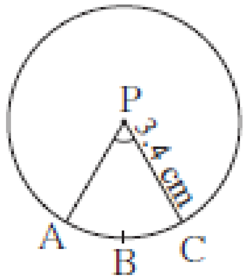


Fig. 7.32

Answer : Radius of circle, r = 3.4 cm

Perimeter of sector, P = 12.8

⇒ P = length of arc + 2 × radius

⇒ Length of arc, l = P – 2×r

⇒ l = 12.8 – 2(3.4)

⇒ l = 6 cm

Let the ∠ APC be θ

As we know that,

$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow l = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow \theta = \frac{360 \times l}{2\pi r}$$

Also,

$$\text{Area of sector, A} = \frac{\theta}{360} \times \pi r^2$$

On substituting the value of theta from above equation,

$$\Rightarrow A = \frac{360 \times l}{360 \times 2\pi r} \times \pi \times r^2$$

$$\Rightarrow A = \frac{l \times r}{2}$$

$$\Rightarrow A = \frac{6 \times 3.4}{2}$$

⇒ A = 10.2 sq. cm

∴ Area of Sector is 10.2 sq. cm

Q. 8. In figure 7.33 O is the centre of the sector.∠ ROQ = ∠ MON = 60°. OR = 7 cm, and OM = 21 cm. Find the lengths of arc RXQ and arc MYN. $\left(\pi = \frac{22}{7}\right)$

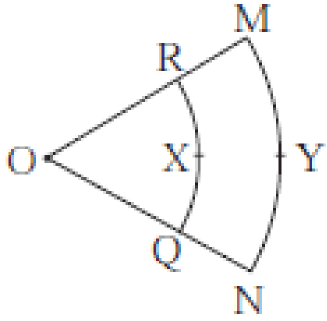


Fig. 7.33

Answer : Let the ∠ ROQ= ∠ MON = θ = 60°

As we know that,

$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow \text{Length(RXQ)} = \frac{60}{360} \times 2\pi \times OR$$

$$\Rightarrow \text{Length(RXQ)} = \frac{1}{6} \times 2 \times \frac{22}{7} \times 7$$

⇒ Length(RXQ) = 7.6 cm

Similarly, $\Rightarrow \text{Length(MYN)} = \frac{60}{360} \times 2\pi \times OM$

$$\Rightarrow \text{Length(MYN)} = \frac{1}{6} \times 2 \times \frac{22}{7} \times 21$$

⇒ Length (MYN) = 22 cm

Q. 9. In figure 7.34, if A(P-ABC) = 154 cm^{2}radius of the circle is 14cm, find

(1) ∠ APC.

(2) l (arc ABC).

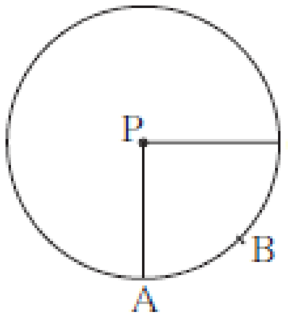


Fig. 7.34

Answer : As we know that,

$$\text{Area of sector, } A = \frac{\theta}{360} \times \pi r^2$$

(1) Let the ∠APC be θ

Radius of circle, r =14 cm

Area of sector, A = 154 cm^2

$$\Rightarrow \theta = \frac{A \times 360}{\pi r^2}$$

$$\Rightarrow \theta = \frac{154 \times 360 \times 7}{22 \times 14^2}$$

⇒ θ = 90°

(2) Since the angle formed is 90°, which is one–fourth of the perimeter of circle

$$\Rightarrow l(\text{arc ABC}) = \frac{1}{4} \times 2\pi r$$

$$\Rightarrow l = \frac{22 \times 14}{2 \times 7}$$

⇒ l = 22 cm

Q. 10. Radius of a sector of a circle is 7 cm. If measure of arc of the sector is –

(I) 30°
(II) 210°

(III) three right angles
Find the area of sector in each case.

Answer : Radius of circle, r = 7cm

(I) Angle subtended by arc, θ = 30°

As we know,

$$\text{Area of sector, A} = \frac{\theta}{360} \times \pi r^2$$

$$A_I = \frac{30}{360} \times \frac{22}{7} \times 7^2$$

⇒ A_I = 12. 83 sq. cm

(II) Angle subtended by arc, θ = 210°

Similarly,

$$\text{Area of sector, } A_{II} = \frac{\theta}{360} \times \pi r^2$$

$$A_{II} = \frac{210}{360} \times \frac{22}{7} \times 7^2$$

⇒ A_{II} = 89.83 sq. cm

(III) Angle subtended by arc, θ = (3× 90)° = 270°

Similarly,

$$\text{Area of sector, } A_{III} = \frac{\theta}{360} \times \pi r^2$$

$$A_{III} = \frac{270}{360} \times \frac{22}{7} \times 7^2$$

⇒ A_{III} = 115.5 sq. cm

Q. 11. The area of a minor sector of a circle is 3.85 cm² and the measure of its central angle is 36°. Find the radius of the circle.

Answer : As we know that,

$$\text{Area of sector, } A = \frac{\theta}{360} \times \pi r^2$$

Given area of sector, A = 3.85 sq.cm

Radius of circle= r

Central angle, θ = 36°

$$\Rightarrow r = \sqrt{\frac{A \times 360}{\pi \times \theta}}$$

On substituting the values, we get,

⇒ r = 3.5 cm

∴ Radius of circle is 3.5 cm

Q. 12. In figure 7.35, □PQRS is a rectangle. If PQ = 14 cm, QR = 21 cm, find the areas of the parts x, y and z.

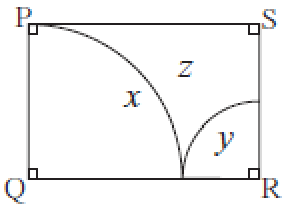


Fig. 7.35

Answer : Since part x is a sector of a circle with radius, r = 14 cm and the central angle is 90°, so the area of x will be equal to one-fourth of the area of circle with PQ as radius.

Area of circle with PQ as radius = π $(PQ)^2$

$$\Rightarrow x = \frac{1}{4} \times \pi \times PQ^2$$

$$\Rightarrow x = \frac{1}{4} \times \frac{22}{7} \times 14^2$$

⇒ x = 154 sq. cm

Similarly, area y is also equal to one-fourth od area of circle with radius, r = QR – PQ

$\Rightarrow$ r = 21 – 14 = 7 cm

Area of circle with r as radius = $\pi (r)^2$

$$\Rightarrow y = \frac{1}{4} \times \pi \times r^2$$

$$\Rightarrow y = \frac{1}{4} \times \frac{22}{7} \times 7^2$$

$\Rightarrow$ y = 38.5 sq. cm

Also,

z = Area of rectangle(PQRS) – x – y

Area of rectangle = PQ × QR

$\Rightarrow$ Area of rectangle = 14 × 21 = 294 sq. cm

$\Rightarrow$ z = 294 – 154 – 38.5

$\Rightarrow$ z = 101.5 sq.cm

∴ the area of x, y and z are 154 sq.cm, 38.5 sq.cm and 101.5 sq.cm respectively

Q. 13. Δ LMN is an equilateral triangle. LM = 14 cm. As shown in figure, three sectors are drawn with vertices as centre and radius7 cm. Find,

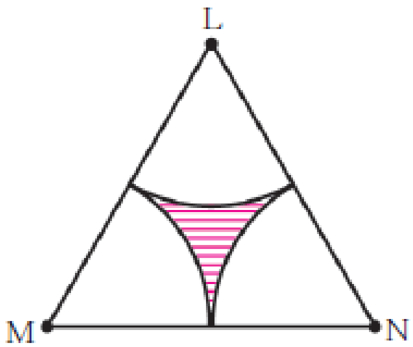


Fig. 7.36

(1) A (Δ LMN)
(2) Area of any one of the sectors
(3) Total area of all three sectors
(4) Area of shaded region

Answer : (1) Side of triangle = LM = a = 14 cm

Since Δ LMN is an equilateral triangle, so the area of the triangle is given by:

$$\text{Area of triangle, } A_T = \frac{\sqrt{3}}{4}a^2$$

$$\Rightarrow A_T = \frac{\sqrt{3}}{4} \times 14^2$$

⇒ A_T = 84.87 sq.cm

(2) Angle subtended by the corner = θ = 60°

As we know,

$$\text{Area of sector, } A_S = \frac{\theta}{360} \times \pi r^2$$

Here $r = \frac{a}{2}$,

$$\Rightarrow A_S = \frac{60}{360} \times \frac{22}{7} \times \left(\frac{a}{2}\right)^2$$

⇒ A_S = 25.67 sq. cm

(3) Total area of all sector, A_{TS} = 3× A_S

⇒ A_{TS} = 3× 25.67

⇒ A_{TS} = 77.01 sq.cm

(4) Area of shaded region, A_R = Area of triangle – Area of all three sectors

⇒ A_S = A_T - A_{TS}

⇒ A_S = 84.87 – 77.01

⇒ A_S = 7.86 sq. cm

∴ area of shaded region is 7.86 sq. cm

Practice Set 7.4

Q. 1. In figure 7.43, A is the centre of the circle. ∠ ABC = 45° and AC = 7√2 cm. Find the area of segment BXC.

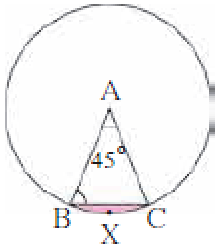


Fig. 7.43

Answer : From the property, we know that, If two sides of a triangle are equal then their corresponding angles are also equal.

So, as AB = AC,

⇒ ∠ ABC = ∠ ACB = 45°

As the sum of angles of a triangle is equal to 180°

⇒∠ ABC + ∠ ACB + ∠ BAC = 180°
⇒45° + 45° + ∠ BAC = 180°
⇒90° + ∠ BAC = 180°

⇒ ∠ BAC = 90°

$$\text{Area Of } \Delta ABC, A_T = \frac{1}{2} \times AB \times AC$$

$$\Rightarrow A_T = \frac{1}{2} \times (7\sqrt{2})^2$$

⇒ A_T = 49 sq.cm

Area of the sector = one-fourth of a circle

$$A_S = \frac{1}{4} \times \pi \times AC^2$$

$$A_S = \frac{1}{4} \times \frac{22}{7} \times (7\sqrt{2})^2$$

⇒ A_S = 77 sq.cm

Area of the shaded region, $A_R = A_S - A_T$

⇒ $A_R = 77 – 49$

⇒ $A_R = 28$ sq.cm

∴ The area of the shaded region is 28 sq. cm.

Q. 2. In the figure 7.44, O is the centre of the circle. M (arc PQR) = 60° OP = 10 cm.

Find the area of the shaded region. (π = 3.14, √3 = 1.73)

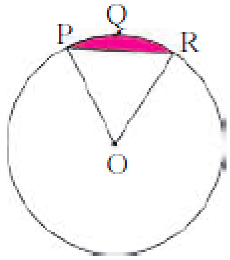


Fig. 7.44

Answer : Since the angle subtended at centre is 60°

And by the property, if two sides of a triangle are equal then their corresponding angles are also equal.

⇒ ∠ ORP = ∠ OPR

As the sum of all internal angles of a triangle is equal to 180°

⇒ ∠ ORP = ∠ OPR = 60°

⇒ Δ OPR is an equilateral triangle.

Area of equilateral triangle, $A_T = \frac{\sqrt{3}}{4}(OP)^2$

⇒ $A_T = \frac{\sqrt{3}}{4} \times 10^2$

⇒ $A_T = 43.25$ sq. cm

Area of Sector (O-PQR), A_S is given as:

$$\text{Area of sector, } A_S = \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow A_S = \frac{60}{360} \times 3.14 \times 10^2$$

$\Rightarrow A_S = 52.33$ sq.cm

Area of shaded region, $A_R = A_S - A_T$

$\Rightarrow A_R = 52.33 - 43.25$

$\Rightarrow A_R = 9.08$ sq.cm

∴ Area of shaded region is 9.08 sq.cm

Q. 3. In the figure 7.45, if A is the centre of the circle. ∠ PAR = 30°, AP = 7.5, find the area of the segment PQR. (π = 3.14)

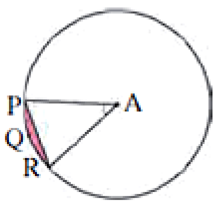


Fig. 7.45

Answer : Radius of circle, r = 7.5 cm

∠ PAR = θ = 30°

Area(A – PQR), A_S:

$$\text{Area of sector, } A_S = \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow A_S = \frac{30}{360} \times 3.14 \times (7.5)^2$$

$\Rightarrow A_S = 14.71$ sq.cm

Also,

$$\text{Area}(\Delta\ APR), A_T = \frac{1}{2} \times r^2 \times \sin(\theta)$$

$$\Rightarrow A_T = \frac{1}{2} \times (7.5)^2 \times \sin(30°)$$

$$\Rightarrow A_T = \frac{1}{2} \times (7.5)^2 \times \frac{1}{2}$$

$\Rightarrow A_T = 14.06$ sq. cm

Area of segment PQR, $A_R = A_S - A_T$

$\Rightarrow A_R = 14.71 – 14.06 = 0.6562$ sq.cm

∴ Area of shaded region is 0.6562 sq.cm

Q. 4. In the figure 7.46, if O is the centre of the circle, PQ is a chord. ∠ POQ = 90°, area of shaded region is 114 cm^2, find the radius of the circle. (π = 3.14)

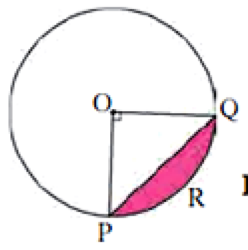


Fig. 7.46

Answer : Area Of shaded region, $A_R = 114$ sq.cm

Area of sector (O-PRQ), A_S = one-fourth of area of circle

$$\Rightarrow A_S = \frac{1}{4}\pi r^2$$

$$\text{Area}(\Delta\ POQ), A_T = \frac{1}{2} \times r^2$$

Area of Shaded Region, $A_R = A_S – A_T$

$$\Rightarrow A_R = \frac{1}{2} r^2 (\frac{\pi}{2} - 1)$$

$$\Rightarrow 114 = \frac{1}{2} \times r^2 \times (\frac{3.14}{2} - 1)$$

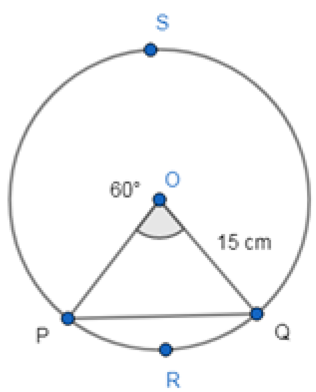
$$\Rightarrow r = \sqrt{\frac{114 \times 2}{0.57}}$$

⇒ r = 20 cm

∴ Radius of the circle is 20 cm

Q. 5. A chord PQ of a circle with radius 15 cm subtends an angle of 60° with the centre of the circle. Find the area of the minor as well as the major segment. (π = 3.14)

Answer :



Radius of circle, r = 15cm

Central angle, θ = 60°

Since the angle subtended at centre is 60°

And by the property, if two sides of a triangle are equal then their corresponding angles are also equal.

⇒ ∠ OQP = ∠ OPQ

As the sum of all internal angles of a triangle is equal to 180°

⇒ ∠ OQP = ∠ OPQ = 60°

⇒ Δ OPQ is an equilateral triangle.

Area of equilateral triangle ΔOPQ , $A_T = \frac{\sqrt{3}}{4}(OP)^2$

$\Rightarrow A_T = \frac{\sqrt{3}}{4} \times 15^2$

$A_T = 97.32$ sq.cm

Area of minor segment, $A_R(PRQ) = \frac{\theta}{360} \times \pi r^2 - A_T$

$\Rightarrow A_R = \left(\frac{60}{360} \times 3.14 \times 15^2\right) - 97.32$

$\Rightarrow A_R = 117.75 – 97.32$

$\Rightarrow A_R = 20.43$ sq.cm

Now,

Area of major segment, $A_S(PSQ) = \pi r^2 - A_R$

$\Rightarrow A_S = (3.14 \times 15^2) - 20.43$

$\Rightarrow A_S = 706.5 – 20.43$

$\Rightarrow A_S = 686.07$ sq.cm

∴ The area of minor segment and major segment is 20.43 sq.cm and 686.07 sq.cm respectively

Problem Set 7

Q. 1. A. Choose the correct alternative answer for each of the following question.

The ratio of circumference and area of a circle is 2:7. Find its circumference.
A. 14π
B. 7/π
C. 7π
D. $\frac{14}{\pi}$

Answer : Circumference of circle, C = $2\pi r$

Area Of circle, A = πr^2

$$\frac{\text{Circumference}}{\text{Area}} = \frac{2}{7}$$

$$\Rightarrow \frac{2\pi r}{\pi r^2} = \frac{2}{7}$$

⇒ r = 7

Circumference = $2 \times r \times \pi$

Circumference = 14π

∴ Option (A) is correct

Q. 1. B. Choose the correct alternative answer for each of the following question.

If measure of an arc of a circle is 160° and its length is 44 cm, find the circumference of the circle.
A. 66 cm

B. 44 cm
C. 160 cm
D. 99 cm

Answer : Measure of arc, θ = 160°

Length of arc, l = 44cm

As we know that,

$$\text{Length of arc, l} = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow 44 = \frac{160}{360} \times \text{Circumference}$$

⇒ Circumference = 99 cm

∴ Circumference of circle is 99cm, Option (D) is correct answer.

Q. 1. C. Choose the correct alternative answer for each of the following question.

Find the perimeter of a sector of a circle if its measure is 90° and radius is 7 cm.
A. 44 cm
B. 25 cm
C. 36 cm
D. 56 cm

Answer : Central angle, θ = 90°

Radius, r = 7cm

As we know that,

$$\text{Length of arc}, l = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow l = \frac{90}{360} \times 2 \times \frac{22}{7} \times 7$$

⇒ l = 11cm

Perimeter of sector =Length of sector + (2 × Radius)

Perimeter = 11 + (2 × 7)

Perimeter = 25cm

∴ Option (B) is correct.

Q. 1. D. Choose the correct alternative answer for each of the following question.

Find the curved surface area of a cone of radius 7 cm and height 24 cm.
A. 440 cm^2
B. 550 cm^2
C. 330 cm^2
D. 110 cm^2

Answer : Radius, r = 7cm

Vertical Height, h = 24 cm

$$\text{Slant Height}, l = \sqrt{r^2 + h^2}$$

$$\Rightarrow l = \sqrt{7^2 + 24^2}$$

⇒ l = 25 cm

Curved Surface Area of cone, A = πrl

$$\Rightarrow A = \frac{22}{7} \times 7 \times 25$$

⇒ A = 550 sq.cm

∴ Option (B) is correct.

Q. 1. E. Choose the correct alternative answer for each of the following question.

The curved surface area of a cylinder is 440 cm^2 and its radius is 5 cm. Find its height.
A. 44/π cm

B. 22π cm
C. 44π cm
D. 22/π cm

Answer : Radius of cylinder, r = 5cm

Height of cylinder = h

Curved Surface Area of cylinder, A = 440 cm^2 = 2πrh

440 = 2× π × 5 × h

$$\Rightarrow h = \frac{44}{\pi}$$

∴ Option (A) is correct.

Q. 1. F. Choose the correct alternative answer for each of the following question.

A cone was melted and cast into a cylinder of the same radius as that of the base of the cone. If the height of the cylinder is 5 cm, find the height of the cone.
A. 15 cm

B. 10 cm
C. 18 cm
D. 5 cm

Answer : Height of cylinder, H = 5 cm

Height of cone =h

Volume of cylinder, V = $\pi r^2 H$

$$\text{Volume of cone}, v = \frac{1}{3}\pi r^2 h$$

Since cone is melted to form cylinder,

$\Rightarrow$ V = v

$$\Rightarrow H = \frac{h}{3}$$

$\Rightarrow$ h = 3 × H

$\Rightarrow$ h = 3 × 5

$\Rightarrow$ h = 15 cm

∴ Option (A) is correct

Q. 1. G. Choose the correct alternative answer for each of the following question.

Find the volume of a cube of side 0.01 cm.
A. 1 cm^3
B. 0.001 cm^3
C. 0.0001 cm^3
D. 0.000001 cm^3

Answer : Side of the cube, a = 0.01 cm

Volume of cube, V = a^3

$\Rightarrow$ V = $(0.01)^3$

$\Rightarrow$ V = 0.000001 cm^3

∴ Option (D) is correct

Q. 1. H. Choose the correct alternative answer for each of the following question.

Find the side of a cube of volume 1 m^3.

A. 1 cm

B. 10 cm
C. 100 cm
D. 1000 cm

Answer : Let the side of cube be ‘a’ cm

Volume of cube, $V = a^3$

⇒ As $1m^3 = 10^6$ cm3

⇒ $a^3 = 10^6$

⇒ a = 100 cm

∴ Option (B) is correct

Q. 2. A washing tub in the shape of a frustum of a cone has height 21 cm. The radii of the circular top and bottom are 20 cm and 15 cm respectively. What is the capacity of the tub? $\left(\pi = \frac{22}{7}\right)$

Answer : Height of frustum, h = 21 cm

Two radii of frustum are, r_1 = 20 cm and r_2 = 15 cm

As we know,

$$\text{Volume of frustum of cone}, V = \frac{1}{3} \times \pi h \times (r_1^2 + r_2^2 + r_1 r_2)$$

$$\Rightarrow V = \frac{1}{3} \times \frac{22}{7} \times 21 \times (20^2 + 15^2 + 20 \times 15)$$

$$\Rightarrow V = \frac{1}{3} \times \frac{22}{7} \times 21 \times 925$$

⇒ V = 20350 cm^3

⇒ V = 20.35 litres

∴ Capacity of bucket is 20.35 litres

Q. 3. Some plastic balls of radius 1 cm were melted and cast into a tube. The thickness, length and outer radius of the tube were 2 cm, 90 cm and 30 cm respectively. How many balls were melted to make the tube?

Answer : Volume of 1 plastic ball = V_B

Radius of ball, r_B = 1 cm

$$V_B = \frac{4}{3} \times \pi r_B^3$$

$$V_B = \frac{4}{3} \times \pi$$

Length of tube, h = 90 cm

Outer radius of tube, r_O = 30 cm

Thickness of tube = 2 cm

Inner radius, r_I = 30 – 2 = 28 cm

Volume of tube, V_T = Volume of Outer cylinder – Volume of inner cylinder

$$V_T = \pi h(r_O^2 - r_I^2)$$

⇒ $V_T = 90\pi(30^2 - 28^2)$

⇒ $V_T = 90 \times 116 \times \pi$

Let 'n' be the number of balls melted.

⇒ $V_T = n \times V_B$

$$\Rightarrow 90 \times 116 \times \pi = n \times \frac{4}{3} \times \pi$$

⇒ n = 7830 balls

∴ 7830 balls were melted to make the tube.

Q. 4. A metal parallelepiped of measures 16 cm × 11cm × 10 cm was melted to make coins. How many coins were made if the thickness and diameter of each coin was 2 mm and 2 cm respectively? $\left(\pi = \frac{22}{7}\right)$

Answer : Volume of parallelepiped, V_P = Product of its three dimensions

⇒ $V_P = 16 \times 11 \times 10$

⇒ $V_P = 1760 \text{ cm}^3$

Thickness of coin, t = 2mm = 0.2 cm

Diameter of coin = 2 cm

⇒ Radius of Coin, r = 1 cm

Volume of one coin, $V_C = \pi r^2 t$

$$\Rightarrow V_C = \frac{22}{7} \times 1^2 \times \frac{2}{10}$$

⇒ V_C = 0.6285 cubic cm

Let the number of coins made be 'n'

⇒ $V_P = n \times V_C$

$$\Rightarrow n = \frac{1760}{0.6285}$$

⇒ n = 2800

∴ 2800 coins are made.

Q. 5. The diameter and length of a roller is 120 cm and 84 cm respectively. To level the ground, 200 rotations of the roller are required. Find the expenditure to level the ground at the rate of Rs. 10 per sq. m

Answer : Diameter of roller = 120 cm

⇒ Radius, r = 60 cm

Length of Roller, h = 84 cm

$$\text{Curved Surface area of Roller}, A_R = 2\pi rh$$

$$A_R = 2 \times \frac{22}{7} \times 60 \times 84$$

⇒ A_R = 31680 sq. cm

Total area of ground, $A_G = 200 \times A_R$

⇒ A_G = 6336000 cm^2

As 1 $m^2 = 10^4$ cm^2

⇒ A_G = 633.6 sq.m

Rate to level the ground = Rs. 10 per sq.m

$$\text{Total Expenditure} = (\text{Total area of ground}, A_G) \times \text{Rate}$$

⇒ Total Expenditure = 633.6 × 10

⇒ Total expenditure = Rs. 6336

∴ Total expenditure to level the road will be Rs. 6336

Q. 6. The diameter and thickness of a hollow metals sphere are 12 cm and 0.01 m respectively. The density of the metal is 8.88 gm per cm^3. Find the outer surface area and mass of the sphere.

Answer : Thickness of sphere, t = 0.01m = 1 cm

Diameter of sphere = 12 cm

⇒ Outer Radius of sphere, R_O = 6 cm

⇒ Inner Radius of sphere, $R_I = R_O - t = 6 - 1 = 5$ cm

Outer Surface area of sphere, $A = 4\pi R_O^2$

⇒ $A = 4 \times 3.14 \times 6^2$

⇒ A = 452.16 sq.cm

$$\text{Volume of hollow sphere}, V = \frac{4}{3} \times \pi \times (R_O^3 - R_I^3)$$

$$\text{Volume of hollow sphere, } V = \frac{4}{3} \times 3.14 \times (6^3 - 5^3)$$

$$\text{Volume of hollow sphere, } V = \frac{4}{3} \times 3.14 \times (216 - 125)$$

⇒ $V = 381.33\ cm^3$

As Mass = Density × Volume

⇒ Mass = 8.88 × 381.33

⇒ Mass = 3385.94 gm

∴ Outer surface area and mass of the sphere is 452.16 sq.cm and 3385.94 gm

Q. 7. A cylindrical bucket of diameter 28 cm and height 20 cm was full of sand. When the sand in the bucket was poured on the ground, the sand got converted into a shape of a cone. If the height of the cone was 14 cm, what was the base area of the cone?

Answer : Diameter of cylinder = 28 cm

⇒ Radius of cylinder, R = 14 cm

Height of cylinder, H = 20 cm

Volume of cylinder, $V_C = \pi R^2 H$

$$V_C = \frac{22}{7} \times 14^2 \times 20$$

Height of cone, h = 14 cm

Radius of cone = r

$$\text{Volume of cone, } v = \frac{1}{3} h \times \pi r^2$$

Now, as the sand in cylinder forms the cone,

⇒ V = v

$$\Rightarrow \pi R^2 H = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow r = \sqrt{\frac{3 \times R^2 \times H}{h}}$$

$$\Rightarrow r = \sqrt{\frac{3 \times 14^2 \times 20}{14}}$$

⇒ r = 28.98 cm

Base area = πr^2

$$\Rightarrow \text{Base area} = \frac{22}{7} \times (28.98)^2$$

⇒ Base area = 2640 sq. cm

∴ The base area of the cone formed is 2640 sq. cm

Q. 8. The radius of a metallic sphere is 9 cm. It was melted to make a wire of diameter 4 mm. Find the length of the wire.

Answer : Radius of sphere, R = 9 cm

$$\text{Volume Of sphere}, V = \frac{4}{3}\pi R^3$$

Let the length of wire be H

Radius Of wire, r = 2 mm = 0.2 cm

Volume of wire, v = $\pi r^2 H$

As sphere is melted to make a wire,

⇒ V = v

$$\frac{4}{3}\pi R^3 = \pi r^2 H$$

$$\Rightarrow H = \frac{4}{3} \times \frac{R^3}{r^2}$$

$$\Rightarrow H = \frac{4}{3} \times \frac{9^3}{0.2^2}$$

⇒ H = 24300 cm

⇒ H = 243 m

∴ The length of the wire formed is 243 m

Q. 9. The area of a sector of a circle of 6 cm radius is 15π sq.cm. Find the measure of the arc and length of the arc corresponding to the sector.

Answer : Let the measure of arc be θ

Area of sector = 15π

Radius, r = 6 cm

As we know,

$$\text{Area of sector, } A_S = \frac{\theta}{360} \times \pi r^2$$

$$15\ \pi = \frac{\theta}{360} \times \pi \times 6^2$$

⇒ θ = 150°

Also,

$$\text{Length of arc, } l = \frac{\theta}{360} \times 2\pi r$$

$$\text{Length of arc, } l = \frac{150}{360} \times 2 \times \pi \times 6$$

⇒ l = 5π

∴ The length of the arc is equal to 5π.

Q. 10. In the figure 7.47, seg AB is a chord of a circle with centre P. If PA = 8 cm and distance of chord AB from the centre P is 4 cm, find the area of the shaded

portion.

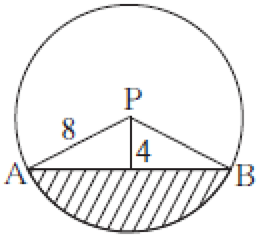


Fig.7.47

Answer : Let ∠ APB = θ

Therefore, $\cos(\frac{\theta}{2}) = \frac{\text{base}}{\text{hypotenuse}}$

$$\cos(\frac{\theta}{2}) = \frac{\text{Distance of chord from centre}}{\text{AP}}$$

$$\Rightarrow \cos(\frac{\theta}{2}) = \frac{4}{8}$$

$$\Rightarrow \cos(\frac{\theta}{2}) = \frac{1}{2}$$

⇒ θ = 120°

$$\text{Area of } \Delta\text{APB}, A_T = \frac{1}{2} \times (\text{AP})^2 \times \sin(\theta)$$

$$\text{Area of } \Delta\text{APB}, A_T = \frac{1}{2} \times (8)^2 \times \sin(120)$$

$$\text{Area of } \Delta\text{APB}, A_T = \frac{1}{2} \times 64 \times \frac{\sqrt{3}}{2}$$

⇒ A_T = 27.68 sq.cm

Also,

$$\text{Area of sector}, A_S = \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow A_S = \frac{120}{360} \times 3.14 \times 8^2$$

$\Rightarrow A_S = 66.96$ sq.cm

Area of shaded region, $A_R = A_S - A_T$

$\Rightarrow A_R = 66.96 – 27.68$

$\Rightarrow A_R = 39.28$ sq. cm

∴ area of shaded region is 39.28 sq. cm

Q. 11. In the figure 7.48, square ABCD is inscribed in the sector A- PCQ. The radius of sector C - BXD is 20 cm. Complete the following activity to find the area of shaded region.

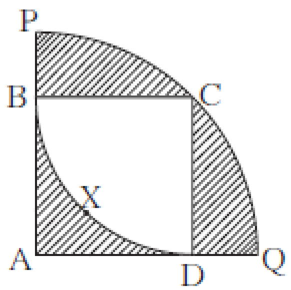


Fig. 7.48

Answer : Side of square ABCD = radius of sector C - BXD = 20 cm

Area of square = $(\text{side})^2 = 200^2 = 400$ sq.cm

Area of shaded region inside the square = Area of square ABCD - Area of sector C - BXD

$$\Rightarrow 400 - \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow 400 - \frac{90}{360} \times 3.14 \times 400$$

$$\Rightarrow 400 - 314$$

⇒ 76 sq.cm

Radius of bigger sector = Length of diagonal of square ABCD

$\Rightarrow 20\sqrt{2}$

Area of the shaded regions outside the square = Area of sector A - PCQ - Area of square
ABCD = A(A - PCQ) - A(ABCD)

$$\Rightarrow \frac{\theta}{360} \times \pi \times r^2 - 400$$

$$\Rightarrow \frac{90}{360} \times 3.14 \times (20\sqrt{2})^2 - 400$$

$\Rightarrow$ 628 – 400

$\Rightarrow$ 228 sq.cm

∴ total area of the shaded region = 86 + 228 = 314 sq.cm.

Q. 12. In the figure 7.49, two circles with centre O and P are touching internally at point A. If BQ = 9, DE = 5, complete the following activity to find the radii of the circles.

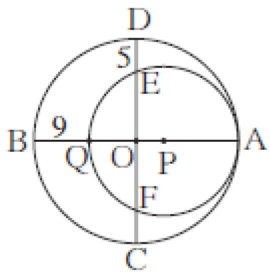


Fig. 7.49

Answer : Let the radius of the bigger circle be R and that of smaller circle be r.

OA, OB, OC and OD are the radii of the bigger circle

∴ OA = OB = OC = OD = R

PQ = PA = r

OQ = OB -BQ =R – 9

OE = OD - DE = R – 5

As the chords QA and EF of the circle with centre P intersect in the interior of the circle, so by the property of internal division of two chords of a circle,

OQ × OA = OE × OF

(R- 9) × R = (R- 5) × (R - 5) ……… (∵ OE = OF)

R^2 - 9R = R^2 - 10R + 25

R = 25

AQ = 2r = AB – BQ

2r = 50 – 9 = 41

$$r = \frac{41}{2} = 20.5$$